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Energimyndighetens titel på projektet – engelska *INTEGRATE!GEO –Next generation tools of large-scale borehole fields in thermal networks*	
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Foreward

The project has been financed within the program *Termo – Värme och Kyla för framtidens energisystem*. Furthermore, the project has highly benefited from the collaboration with prof. Massimo Cimmino from Polytechnique Montréal.

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Sammanfattning

Borrhålsfält är nyckelkomponenter för framtidens energisystem eftersom de kan användas som säsongsbetonad termisk lagring och dra nytta av mycket stora mängder tillgänglig termisk energi som annars skulle gå till spillo.

Integrationen av borrhålsfält i energisystemet och deras optimering är dock ett icke-trivialt problem och det finns många potentiella sätt att driva sådana system.

Nuvarande verktyg för borrhålsmodellering har fortfarande begränsningar som hindrar utvärdering och optimering av driftsstrategier som seriedrift av borrhål eller partialisering av borrhålsfält. Detta projekt syftade till att utveckla verktyg för modellering av borrhålsnätverk och göra dem mer flexibla jämfört med nuvarande teknik, samtidigt som de är effektiva och lämpliga för att studera integrationen av borrhålsfält i energisystemet.

Under projektet uppnåddes grundläggande förbättringar i skalbarheten hos de beräkningsalgoritmer som används. Denna viktiga utveckling minskade avsevärt den beräkningstid som krävs av borrhålsmodeller och gjorde långsiktig timsimulering med detaljerad beskrivning av borrhålsnätverkets drift möjlig. Simuleringstester av den detaljerade driften av stora borrhålsfält med komplexa operationer (t.ex. borrhål som drivs i serie) presenteras för att visa upp den flexibilitet och effektivitet som uppnåtts. Modellerna har implementerats i verktyg med öppen källkod som är fritt tillgängliga för forskare och ingenjörer inom geotermisk teknik.

Summary

Borehole fields are key components for the energy system of the future as they can be utilized as seasonal thermal storage and take advantage of very large quantities of available thermal energy that would be otherwise wasted.

Integration of borehole fields in the energy system and their optimization is however a non-trivial problem and there exists a lot of potential ways to operate such systems. Current tools for borehole modeling have still limitations that hinder the evaluation and optimization of operational strategies such as borehole series operation or borehole field partialization. This project aimed at developing borehole network modeling tools and make them more flexibile compared to current state of the art, while being performant and suitable for studying the integration of borehole fields in the energy system.

During the project, fundamental improvements were achieved in the scalability of the computational algorithms utilized. This key development vastly reduced the computational time required by borehole models and made long term hourly simulation with detail description of the borehole network operation feasible. Simulation tests of the detailed operation of a large borehole fields with complex operations (e.g. borehole operated in series) are presented to showcase the flexibility and efficiency achieved. The models have been implemented in open source tools freely available for scientist and engineers in the field of geothermal technology.

Introduction/Backgrund

Thermal networks combined with seasonal heat storage represent a highly promising approach for the effective integration of waste heat and renewable heat sources into the energy system. Effective utilization could significantly reduce the consumption of primary energy for space heating and cooling in buildings and have a very large impact for the society from an economical, environmental and security of supply perspective.

Figure 1 Schematics of systems and energy sources connected to a borehole field through thermal networks. shows an example of a set of systems that could be integrated in a thermal network to optimize the energy flows and reduce the overall primary energy needs. In the example, heat pumps and solar introduce loads that have signficant variation during the day. On the other hand, the dynamics of the borehole field present different time constants: from a few hours, relative to individual borehole operations, to months and up to several years related to the thermal interaction between neighboring boreholes. Such coupled systems are as result inherently complex, both in their design and in their operation. Fully exploiting their potential requires advanced forecasting tools yielding sensible predictions for both short term operation (few hours and days) and long term operation (life span of the installation).

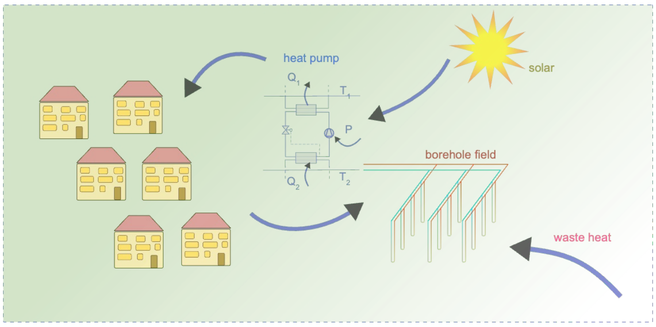


Figure 1 Schematics of systems and energy sources connected to a borehole field through thermal networks.

In this context, modeling and simulation play a central role, as they enable the evaluation of operational strategies and system behavior within virtual environments before implementation highly reducing costs and risk.

There is a vast literature in this field and several researcher have worked on this problem. Key contributions are in the seminal work carried out by Claesson, Bennet, Eskilsons and Hellström [1-6] who posed the scientifical basis behind most modeling tools used in the field. Notable contributions in recent years have been also provided by Cimmino, Lazzarotto, Bernier, Spitler, Lamarche and many other authors [7-19].

Nevertheless, limitations of existing modeling approaches still restrict the insights they can provide to solve relevant questions regarding borefield network operations. In particular there is currently no dedicated available standard tool that allows to perform long-term simulation (e.g. 20 years) with fine temporal resolution (e.g. hourly) and study complex borefield operation such as partialization of the borehole field or arbitrary piping network topologies such as loops of series connected boreholes (Figure 2). The ability to accurately simulate these complex operations is key to fully exploit the potential of large borehole field for seasonal heat storage where loads can vary significantly and simple borehole field operation might not be the ideal choice to optimize thermal and hydraulic performance of the system.

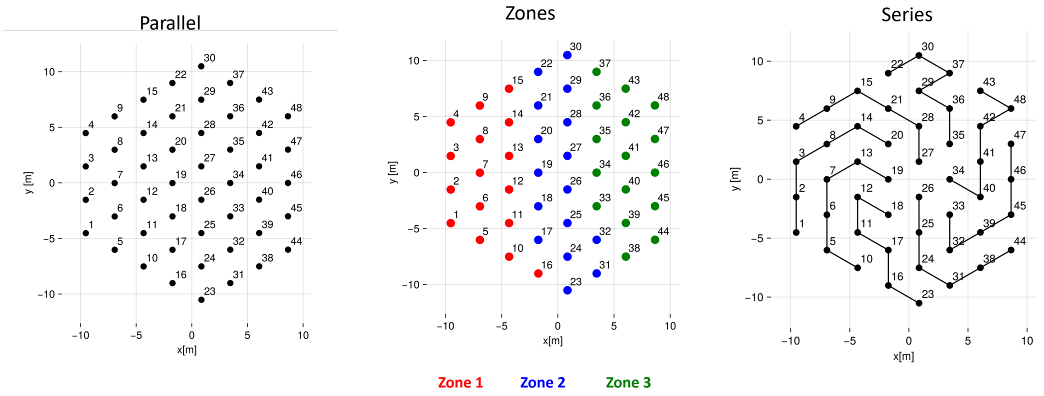


Figure 2 Illustration of how the same borehole field layout can be connected hydraulically in different ways. Left figure: a manifold supplies all the boreholes in the field yielding a parallel operation (left figure). Central figure: a manifold supplies groups of boreholes and each group of boreholes could be controlled independently through valves or pumps (zone operation). Right figure: the manifold supplies loops of boreholes connected in series.

In this project, the approach used to tackle this problem is based on the work of Lazzarotto [15] that introduced the idea of coupling a semi-analytical borehole field model with constrains arising from the topology of the network and solve the coupled problem sequentially at eact time step of the simulation. The benefit of this method is that it can capture at any given time how heat is distributed in the borehole field based on local conditions, i.e. the fluid temperature supplied to each individual borehole and the current temperature of the ground in their surroundings. These conditions are the key constraints to achieve representative simulation of the problem. While being very effective and promising, the network-based approach was applied so far mostly for monthly time-step and did not scale well in terms of computational time for finer time resolutions.

The goal set in the project was therefore working on this fundamental bottleneck and improve the time-superposition algorithms used in the semi-analytical scheme. Time-superposition allows to handle simulations with thermal loads that change in time according to arbitrary functions. This feature makes the method flexible and suitable for realistic simulations. The load is typically modeled as a stepwise constant function, and the overall effect of the load history is simply the sum of the contribution of the loads corresponding to each previous time step. This is a simple idea, but its naïve implementation is plagued by a scaling problem which hinders its use for long term simulation with high temporal resolutions, e.g. hourly simulations with 20 years horizon. The algorithmic complexity of this approach in fact scales with squares of the number of timesteps $O(N_t^2)$. This means that the time to perform the computation is highly influenced by the choice of time-resolution. As an example, if we increase the resolution from daily to hourly the increase in number of operations required is approximately proportional to $24^2 = 576$ (where 24 is the number of hours in a day). This example shows how the increase in time resolution can quickly impact the computational time by order of magnitudes and quickly make simulations unfeasible as resolution increases.

The two acceleration strategies found in literature that tackle this problem are the *load aggregation* scheme [6-8] and the so-called *non-history method* approach [18-19]. In the project both methods were explored and contributions to both ideas were developed and discussed in reaserch articles.

Given the nature of the topic, the implementation and part of the result sections of this report are technical and focused on the work done of algorithms improvements. The presentation provided intends to be focused on key ideas and results, leaving a lot of details to the relative scientific publications. The final part of the result section provides an example of usage of a borehole network model to showcase what can be achieved in terms of functionality and performance using the methodology developed.

The technical developments of this project have been implemented in research software publicly available on the platform Github. The development of the computational tools is a co-development carried out together with project RECOIN (P2022-01040).

Implementation

The implementation of the project can be divided into to parts.

1. Fundamental developments of computational algorithms to accelerate borehole simulations.
2. Implementation of technical developments in computer software.

Acceleration algorithms

Time-superposition acceleration methods

The time-superposition's scaling issue is a well-known issue for the discrete convolution problem arising in semi-analytical borehole models and two relevant approaches to address it can be found in literature: the *load aggregation schemes* and the *non-history acceleration*. In the following sub-sections, a short presentation of the relevant background information and the key ideas explored in the project to improve on the existing method are given.

Optimization of load aggregation considering slow and fast dynamics in a borehole field.

Load aggregation techniques are currently the most common acceleration approach in practice. The method consists in organizing the load history into blocks where each block can comprise several time-steps, assigning only one load value per-block, and applying the time superposition procedure to the load blocks instead of the complete load history. This approach yields a significant reduction in the number of loads that needs to be considered in the superposition ($N_{blocks} << N_t$ where N_t is the number of time steps), and strongly reduces the computation required for this operation. The blocks are designed to be very small for recent loads and increasingly large for older loads. This design allows to retain high resolution information on the recent load history and increasingly compress information for older loads. The block-load history update mechanism taking place at each timestep consists in transfering heat from one block to the following one, and it is designed to enforce energy conservation.

In order to improve on this method in the context of borehole network simulations, we decided to leverage the fact that the borehole fields have *fast dynamics* and *slow dynamics*. Fast dynamics can only be attributed to the effect that boreholes have on themselves. For self-responses it is therefore necessary to evaluate the effect with the finest time resolution desired. On the other hand, the mutual interaction between neighboring boreholes only affects slow dynamics. This means that the evaluation of this contribution, which is the largest share in terms of computation, can be performed at lower frequency without significant loss of accuracy and with a significant reduction in computational cost.

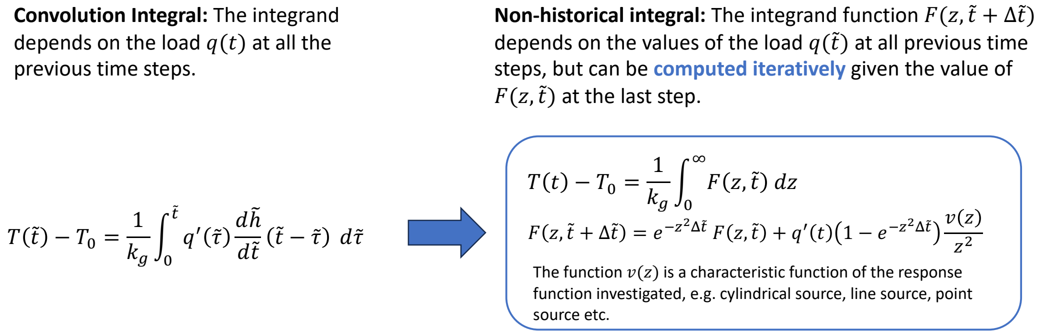
The work is discussed in the article:

M. Cimmino, A. Lazzarotto. Load aggregation for thermal interactions between boreholes. *IGSHPA Research Conference Montréal*, May 28-30, 2024.

Advances in the non-history acceleration algorithm

The *non-history* method was introduced by Lamarche [18-19] and provides a very different strategy to compress the load history compared to the load aggregation scheme. The method essentially manipulates the starting problem of time-superposition, which in continuous form is the convolution integral, and transforms it into an equivalent integral, the “non-history” integral.

The method is a time marching scheme which is a key feature for integration with building and energy systems in co-simulations environment. As it is showed in Figure 3, the updates of the integrand function of the non-history integral can be performed iteratively and the problem does not grow in size as the number of time steps increases. Furthermore, as opposed to the load aggregation method, the non-history integral is not an approximation but is exactly equivalent to the convolution integral. This means that the error introduced by this method boils down exclusively to the accuracy of the numerical integration of the non-history integral and is not related to the load compression strategy.



Convolution Integral: The integrand depends on the load $q(t)$ at all the previous time steps.

$$T(\tilde{t})-T_0=\frac{1}{k_g}\int_0^{\tilde{t}} q'(\tilde{\tau})\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t}-\tilde{\tau})\ d\tilde{\tau}$$

Non-historical integral: The integrand function $F(z,\tilde{t}+\Delta\tilde{t})$ depends on the values of the load $q(\tilde{t})$ at all previous time steps, but can be **computed iteratively** given the value of $F(z,\tilde{t})$ at the last step.

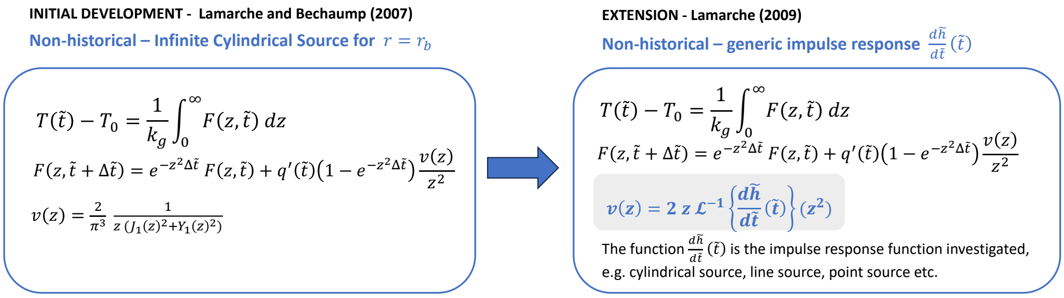
$$T(t)-T_0=\frac{1}{k_g}\int_0^{\infty} F(z,\tilde{t})\ dz$$

$$F(z,\tilde{t}+\Delta\tilde{t})=e^{-z^2\Delta\tilde{t}}\,F(z,\tilde{t})+q'(t)\left(1-e^{-z^2\Delta\tilde{t}}\right)\frac{v(z)}{z^2}$$

The function $v(z)$ is a characteristic function of the response function investigated, e.g. cylindrical source, line source, point source etc.

Figure 3 Illustration of convolution integral and non-historical integral.

The development of the original non-history method was done in two steps (Figure **4**). In the first step Lamarche develop the analytical formulation for the specific case of response of an infinite cylindrical source at the radius r=r$_b$ [18]. In a second study, he further extended the work for generic response functions e.g. line sources, g-functions, etc [19]. The key result in the second work was showing that the characteristic function *v*(*z*) (*z* is the integration domain) of the response function investigated could be computed for any generic impulse response in time domain *dh/dt*(*t*).



INITIAL DEVELOPMENT - Lamarche and Bechaump (2007)

Non-historical – Infinite Cylindrical Source for $r=r_b$

$$T(\tilde{t})-T_0=\frac{1}{k_g}\int_0^{\infty} F(z,\tilde{t})\ dz$$

$$F(z,\tilde{t}+\Delta\tilde{t})=e^{-z^2\Delta\tilde{t}}\,F(z,\tilde{t})+q'(\tilde{t})\left(1-e^{-z^2\Delta\tilde{t}}\right)\frac{v(z)}{z^2}$$

$$v(z)=\frac{2}{\pi^3}\,\frac{1}{z\,(J_1(z)^2+Y_1(z)^2)}$$

EXTENSION - Lamarche (2009)

Non-historical – generic impulse response $\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t})$

$$T(\tilde{t})-T_0=\frac{1}{k_g}\int_0^{\infty} F(z,\tilde{t})\ dz$$

$$F(z,\tilde{t}+\Delta\tilde{t})=e^{-z^2\Delta\tilde{t}}\,F(z,\tilde{t})+q'(\tilde{t})\left(1-e^{-z^2\Delta\tilde{t}}\right)\frac{v(z)}{z^2}$$

$$\boldsymbol{v(z)=2\,z\,\mathcal{L}^{-1}\left\{\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t})\right\}(z^2)}$$

The function $\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t})$ is the impulse response function investigated, e.g. cylindrical source, line source, point source etc.

Figure 4 Summary of relevant results discussed in the work of Lamarche.

This work made the method more generic and useful. However, the evaluation of *v*(*z*) required performing the Inverse Laplace Transform of the time domain impulse response *dh/dt*(*t*). This operation was performed numerically using Gaver-Stephen algorithm. This fact is according to the authors a limitation of the method because numerical Inverse Laplace Transform presents a few issues. Firstly, the inverse is ill-posed, not always stable and could result in unknown numerical error in the inverted function. Secondly, not having access to the analytical expression of the inverse limits the understanding and optimization that can be performed to optimize the numerical integration of the *non-history integral* necessary for the evaluation of the temperature response of interest.

The objectives of this work were therefore: evaluating whether it was possible to perform the inverse Laplace Transform of impulse responses to find analytical inverses *v*(*z*) for relevant characteristic functions for borehole applications, improving the understanding of the method, and optimizing the error control and computational performance.

The investigation led to several interesting and novel results and confirmed that a careful implementation of the method can provide accurate and performant results that can be used to highly accelerate simulations involving thermal interaction in borehole fields.

This work is discussed in the article:

A. Lazzarotto, M. Basquens, M. Cimmino. A non-history dependent temporal superposition algorithm for the point source solution, *IGSHPA Research Conference Montréal*, May 28-30,2024https://doi.org/10.22488/okstate.24.000021

A. Lazzarotto, M. Basquens, M. Cimmino. A non-history dependent temporal superposition algorithm for the finite line source solution. *Submitted to Journal*. Preprint available https://doi.org/10.48550/arXiv.2507.18200

M. Basquens, A. Lazzarotto. A causality inspired acceleration method for the fast temporal superposition of the finite line source solutions. *Submitted to Journal*. https://doi.org/10.48550/arXiv.2507.18200

Modeling tools development and testing

The algorithms and ideas investigated in this project have been implemented in open-source research software:

- The load aggregation work has been implemented using pygfunction and geothermsim.
- The non-history algorithm has been implemented and tested in the Github repository https://github.com/marcbasquensmunoz/FiniteLineSource.
- The non-history algorithm has then been integrated in the simulator https://marcbasquensmunoz.github.io/BoreholeNetworksSimulator.jl/dev/. The simulator couples a borehole heat exchanger model, a borehole field interaction model and the hydraulic network topology model.

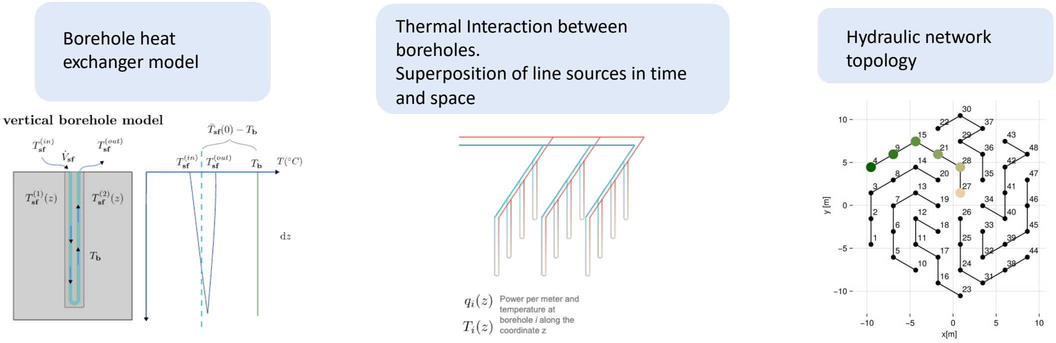


Figure 5 Illustration of the three model components that are coupled within the research software BoreholeNetworksSimulator.jl.

The work on the simulation software tools has been a key step in the implementation plan. First, it allowed testing and measuring in practice the efficacy of the acceleration strategies investigated. Furthermore, the tools utilized are open source and as such openly available. The software can be used and modified by researchers and engineers interested in the topic. This fact makes the technical developments of this project more transparent, repeatable and accessible for the end user.

Results

Long-term fine resolution simulations

Accelerated load aggregation for borehole fields.

An efficient load aggregation approach has been introduced for network-based simulations involving multiple boreholes. The method separates the system dynamics into fast and slow components. Fast dynamics, corresponding to the self-interaction of individual boreholes, are resolved at every timestep using established load aggregation techniques. Slow dynamics, representing thermal interactions between different heat sources, are evaluated less frequently by incorporating a time delay in the update of their calculations.

We introduced a *push-forward* strategy that allows to evaluate the effect of mutual interactions *d* timesteps ahead into the future, and use this result to perform a linear interpolation to quickly evaluate the interaction in the intermediate timesteps.

The error caused by the delayed update depends on the minimum mutual distance between the boreholes in the field and the frequency of updates: the exact thermal interaction for boreholes at larger distances can potentially be evaluated at lower frequency and still achieve acceptable accuracy. Figure 6 shows a relative measure of the error caused in the temperature field using this approximation for the case of a single borehole loaded with a step heat injection. The error caused by the interpolation in this case shows that once we fix a desired tollerance we can find the minimal acceptable delay *d* suitable to evaluate the response at a given distance. This result provides a methodology to evaluate relevant delays *d* for borefields simulations.

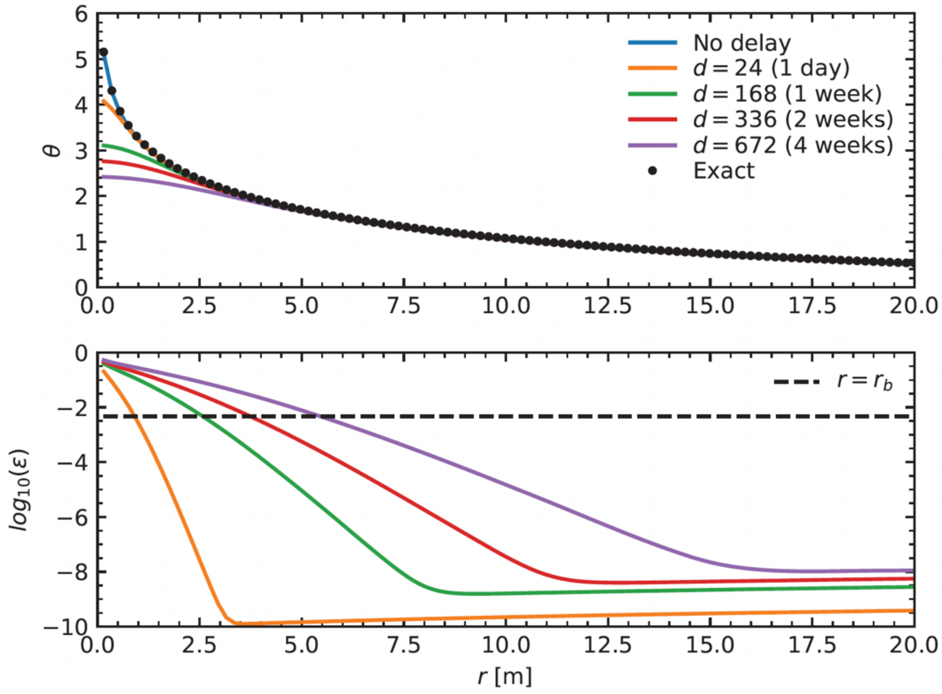


Figure 6 Dimensionless length-averaged temperature profile around borehole $i = 1$ for different time delays d (top), and logarithm of the relative error ε (bottom) evaluated at the 20th year of operation.

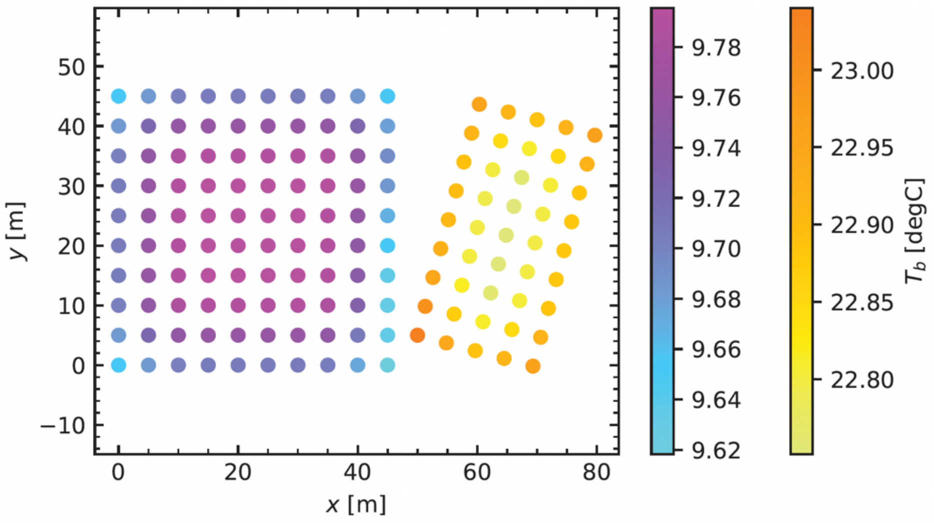


Figure 7 shows the application of the method to the case of two borehole fields operated indipendently. A simulation where all interactions were computed hourly was made to provide a reference to compare with when introducing the approximation. The push-forward strategy was tested with a delay of 336 days to balance efficiency and accuracy for the simulated borehole fields characterized by a minimum boreholes distance of 5 meters.

The 20 years hourly simulation of the two interacting borehole fields shows that a maximum error of 0.003 °C on the inlet fluid temperatures can be obtained while decreasing the calculation time from 275 seconds of the reference case to 26.2 seconds of the approximation.

In summary the methodology introduced provides a strategy to approximate and optimize the evaluation of thermal interactions in the field that can speed up significantly semi-analytical methods and make large borehole field simulation feasible.

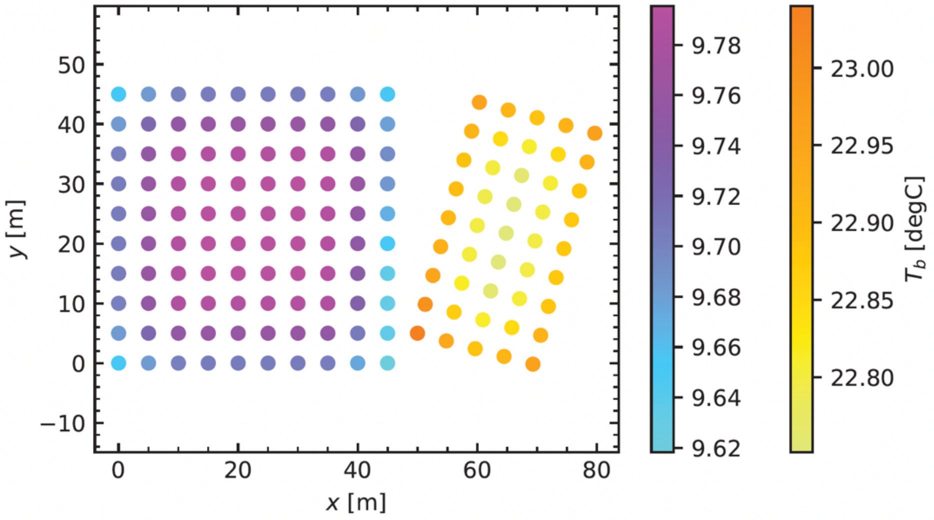


Figure 7 Positions of boreholes in two interacting borehole fields. Colors indicate the borehole wall temperatures at the 8590th hour of the 20th simulation year.

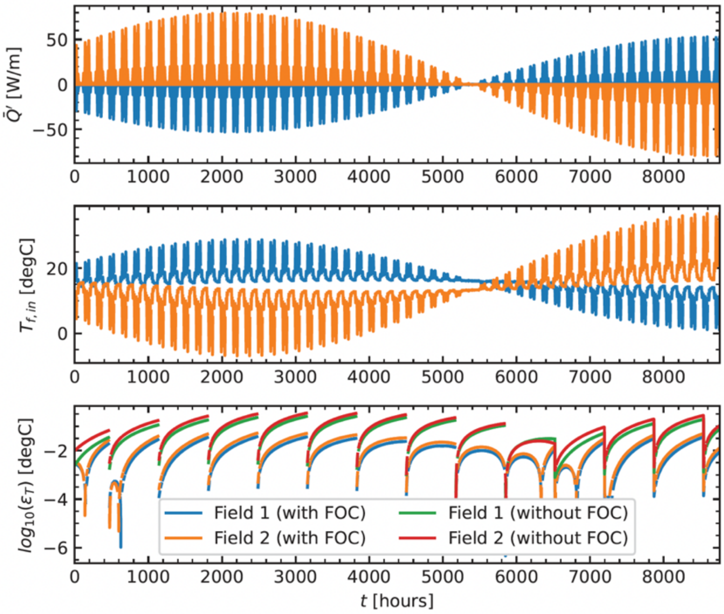
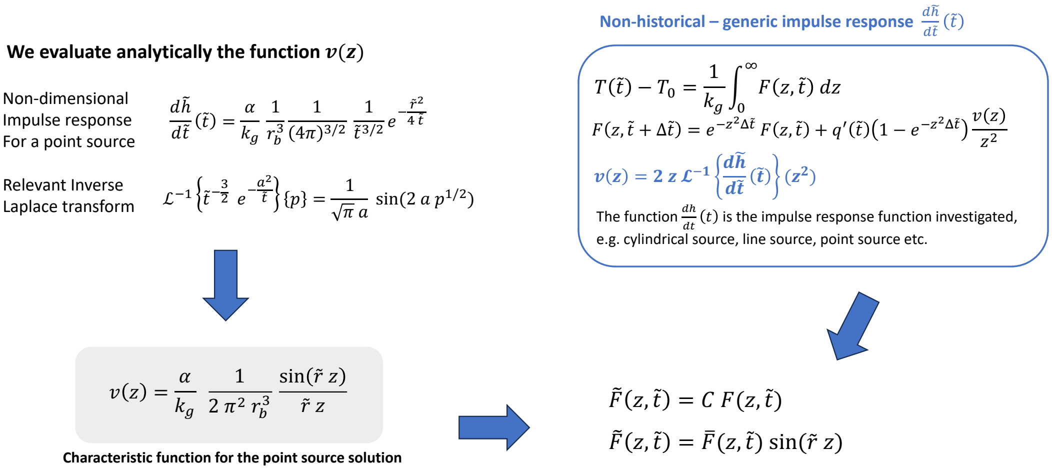


Figure 8 Average heat extraction rates during the 20th simulation year (top), inlet fluid temperatures (middle), and absolute error on the inlet fluid temperature with and without the first order correction (FOC) (bottom).

Non-history development – point source solution

As discussed in the implementation section, the first step to advance the work of Lamarche on the non-history acceleration method was determining if any response function relevant for borehole applications could be Laplace inverted. Attemps to directly invert the finite line source (the natural candidate for borehole applications) were made but they were unsuccessful. The key intuition that allowed to go forward with this approach was focusing on the point source solution. The point source solution is relevant as it can be used to build line sources by means of superposition of the effect in space. It was found that for this response function there exist a simple analytical expression for the inverse Laplace Transform (Figure 9). The resulting function is proportional to the sync function $v(z) \propto \sin(\tilde{r}z)/\tilde{r}z$ which is a highly oscillating function for large values of the non-dimensional radius $\tilde{r} = r/r_b$ (Figure 10). This result shows the benefit of this analytcal approach. Numerical integration techniques, such as Gauss integration, are in fact not suitable for highly oscillating integrand functions and specialized methods are required.



We evaluate analytically the function $v(z)$

Non-dimensional Impulse response For a point source $\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t}) = \frac{\alpha}{k_g}\frac{1}{r_b^3}\frac{1}{(4\pi)^{3/2}}\frac{1}{\tilde{t}^{3/2}}e^{-\frac{\tilde{r}^2}{4\,\tilde{t}}}$

Relevant Inverse Laplace transform $\mathcal{L}^{-1}\left\{\tilde{t}^{-\frac{3}{2}}\,e^{-\frac{a^2}{\tilde{t}}}\right\}\{p\} = \frac{1}{\sqrt{\pi}\,a}\,\sin(2\,a\,p^{1/2})$

$$v(z) = \frac{\alpha}{k_g}\,\frac{1}{2\,\pi^2\,r_b^3}\,\frac{\sin(\tilde{r}\,z)}{\tilde{r}\,z}$$

Characteristic function for the point source solution

Non-historical – generic impulse response $\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t})$

$$T(\tilde{t}) - T_0 = \frac{1}{k_g}\int_0^\infty F(z,\tilde{t})\,dz$$

$$F(z,\tilde{t}+\Delta\tilde{t}) = e^{-z^2\Delta\tilde{t}}\,F(z,\tilde{t}) + q'(\tilde{t})\left(1 - e^{-z^2\Delta\tilde{t}}\right)\frac{v(z)}{z^2}$$

$$\boldsymbol{v(z) = 2\,z\,\mathcal{L}^{-1}\left\{\frac{d\tilde{h}}{d\tilde{t}}(\tilde{t})\right\}(z^2)}$$

The function $\frac{dh}{dt}(t)$ is the impulse response function investigated, e.g. cylindrical source, line source, point source etc.

$$\tilde{F}(z,\tilde{t}) = C\,F(z,\tilde{t})$$

$$\tilde{F}(z,\tilde{t}) = \bar{F}(z,\tilde{t})\,\sin(\tilde{r}\,z)$$

Figure 9 Procedure for the determination of the characteristic functions $v(z)$ for a point source in unbounded domain.

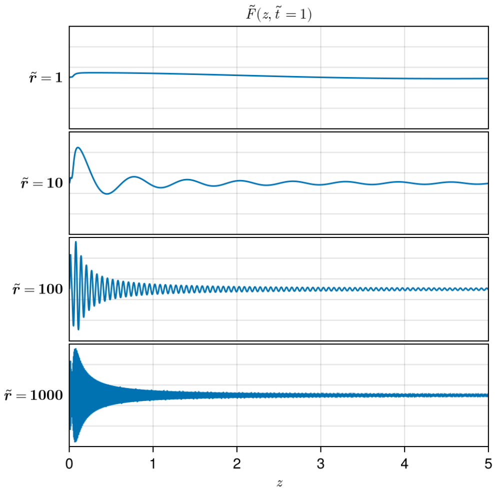


Figure 10 Integrand function for point source arising from the non-history. The integral of this function yields the desired temperature. The integrand function is highly oscillating and the oscillations increase as with the non dimensional radius $\boldsymbol{\tilde{r} = r/r_b}$.

In the article *“A non-history dependent temporal superposition algorithm for the point source solution”* the specialized integration strategy method developed by Backalov Vasil’eva was utilized to solve this issue. The results obtained for the point source showed that it is possible to achieve a precision of around 10^{-15} which is close to machine precision. The overall complexity of the algorithm does not grow with the number of time step.

Non-history acceleration method development – line source solution

The work was then extended to line sources (very important for borehole applications) by numerical integration of the point source result along segments. The article *“A non-history dependent temporal superposition algorithm for the finite line source solution”* presents in details the formulation and testing of this idea. The results show that it is possible to achieve very high precision (10^{-12}) by increasing the number of integration points over the line. Also in this case the algorithm complexity does not grow with the number of steps.

One downside of this method is that the necessary precomputation set up, i.e. the set of operations that needs to be done only once to prepare the time-dependent simulation, is costly and can limit the usage of this methodology for large borehole fields where many finite line sources needs to be considered. This computational cost is due to the evaluation of Bessel’s functions at many points which arises in the choosen numerical method for integration of highly oscillating functions.

Non-history acceleration method development – organization of time and space scales

The issue discussed in the previous subsection was addressed by realizing that a proper *organization of time and space scales* strongly reduces the difficulties encountered in the numerical integration of the non-history integral. This work is discussed in detail in the article *“A causality inspired acceleration method for the fast temporal superposition of the finite line source solutions”*.

The organization is obtained in two steps. First, similarly to load aggregation, the organization of time scales is obtained by dividing the load history into blocks and applying the non-history method to each load block (Figure 11).

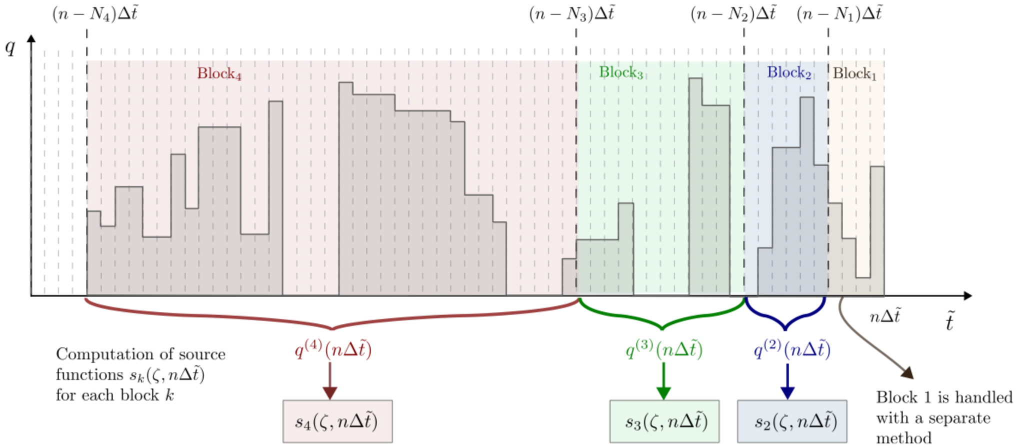
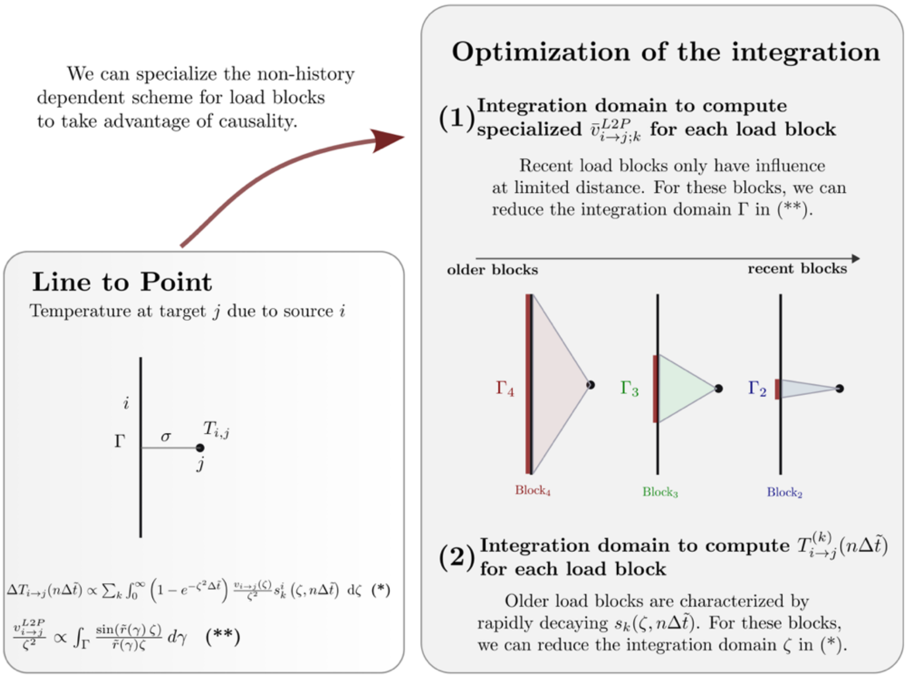


Figure 11 Division of load history into load blocks. The function s_i is the gaussian smoothing of the load and is the compressed version of the load that is utilized in the non-history method.

And second, a further organization of space-time scales is obtained by evaluating the characteristic function v for each interaction and for each load block. The function v is in principle a function that depends only on geometrical parameters and not on time. However, we can consider that, given a step heat source (applied to a point or a finite line), there exists a limited region of influence around it, i.e. a region where there is non-negligible contribution to the temperature field. The region size depends on a choosen tolerance and on how long the heat step has been applied, i.e. time. This means that, when evaluating the function *v* for a given evaluation point, it is not always necessary to consider the contributions of the whole line source. Instead, only the parts that are contributing to the temperature for the given time range of interest should be considered.

Figure 12 exemplifies this idea. For old load blocks (large region of influence), the whole line source needs to be considered in the computation of *v*. However, for recent load blocks, only part of it contributes to the temperature field and only this part should be included in the line integral (small region of influence).

Figure 13 shows the effect of applying this strategy in practice. The integrand function arising from the non-history method when considering only the part of the heat source that actually contributes to the point of interest is very simple and as a result easy to integrate. On the other hand, considering the full line (even if there are part that do no contribute) results in oscillating integrand that requires a lot of integration nodes for accurate evaluation. This result has strong implications that affect how precise and how efficient is the method in evaluating temperature responses.



We can specialize the non-history dependent scheme for load blocks to take advantage of causality.

Line to Point

Temperature at target j due to source i

$\Delta T_{i\to j}(n\Delta\tilde{t}) \propto \sum_k \int_0^\infty \left(1-e^{-\zeta^2\Delta\tilde{t}}\right) \frac{v_{i\to j}(\zeta)}{\zeta^2} s_k^i\left(\zeta, n\Delta\tilde{t}\right)\, \mathrm{d}\zeta$ (*)

$\frac{v^{L2P}_{i\to j}}{\zeta^2} \propto \int_\Gamma \frac{\sin(\tilde{r}(\gamma)\,\zeta)}{\tilde{r}(\gamma)\zeta}\, d\gamma$ (**)

Optimization of the integration

(1) Integration domain to compute specialized $\bar{v}^{L2P}_{i\to j;k}$ for each load block

Recent load blocks only have influence at limited distance. For these blocks, we can reduce the integration domain Γ in (**).

(2) Integration domain to compute $T^{(k)}_{i\to j}(n\Delta\tilde{t})$ for each load block

Older load blocks are characterized by rapidly decaying $s_k(\zeta, n\Delta\tilde{t})$. For these blocks, we can reduce the integration domain ζ in (*).

Figure 12 Schematic representation of the steps used to optimized computations in the non-history method for the case of load blocks.

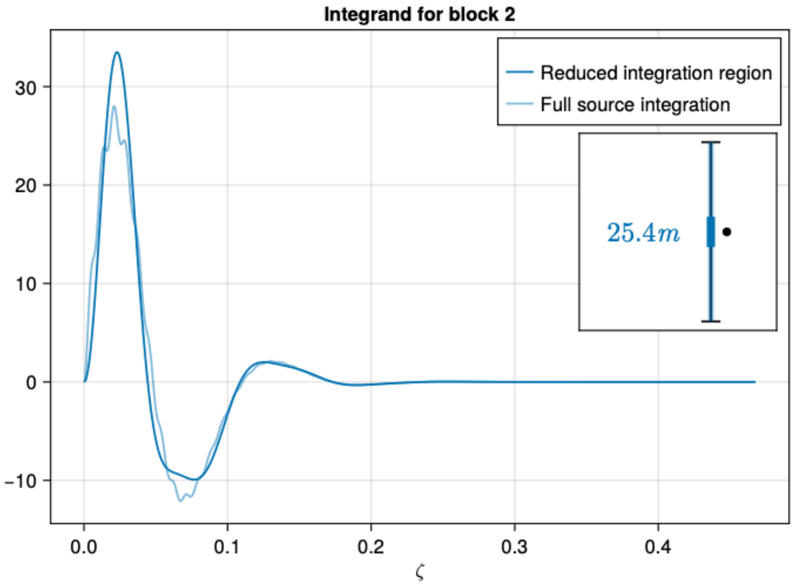


Figure 13 Example of integrand functions obtained by integrating on the full source or on the reduced region based on the region of influence from the article .

Non-history acceleration method – performance and error control

The original approach based on numerical integration of fast oscillating integral and the block method were compared by simulating the effect of a time varying source for 20 years and evaluating the execution time for different error tollerances. The error was evaluated by comparing the result obtained with the non-history method implementations against the discrete convolution computed via FFT algorithm. The geometries considered are point source to point target (P2P), line source to point target (L2P) and line source to line target (L2L).

Table 1 summarizes the computational time relative to the precomputation step, while Table 2 summarizes the computational time relative to the execution of a 20 year simulation with hourly steps (175200 steps). All the simulation were carried out with a laptop equipped with Apple M2 Pro processor.

The results show that the blocks method has significant benefit in terms of precomputation compared to original non-history implementation and reduces the precomputation time by one order of magnitude. On the other hand, in the test carried out, the blocks method introduces an increase in computation for the calculation that needs to be done at each time step. The results presented here are for a single response. All things considered, the blocks method is beneficial in terms of performance in the context of a complete borehole field simulation where several mutual interaction needs to be considered.

Notice that execution time of the approach based on FFT is faster then the non-history method. The FFT method however, is not a time-marching and for this reason is not suitable for the integration in building and energy systems simulations. The comparison with the FFT is made to have a baseline of how the method performs against a very optimized algorithm, but it cannot directlycompared since these two families of methodologies cannot be used for the same application.

Another point that can be observed in Table 1 is the very large time for the precomputation relative to the FFT. The reason for this time is that the response function used in the FFT is evaluated during precomputation for every time step of the simulation and no interpolations are made to speed up the process. This choice is not common in practice when using this technique. However, for this test it was the natural choice since the FFT methodology here is utilized as the reference and using interpolation could introduce unknown error in the results.

Table 1 Execution times of the precomputation for a single interaction between a source and a target, in the original, the blocks, and convolution methods, for several values of the tolerance ϵ and each of the considered cases: point to point, line to point, and line to line.

Precomputation method	$\epsilon = 10^{-2}$	$\epsilon = 10^{-4}$	$\epsilon = 10^{-6}$	$\epsilon = 10^{-8}$	$\epsilon = 10^{-10}$	$\epsilon = 10^{-12}$
Original (P2P)	53 μs	313 μs	575 μs	575 μs	782 μs	911 μs
Blocks (P2P)	125 μs	138 μs	142 μs	155 μs	166 μs	174 μs
Convolution (P2P)	1.4 ms	1.4 ms	1.4 ms	1.4 ms	1.4 ms	1.4 ms
Original (L2P)	592 μs	14 ms	65 ms	80 ms	112 ms	566 ms
Blocks (L2P)	346 μs	772 μs	1 ms	2 ms	4 ms	10 ms
Convolution (L2P)	190 ms	397 ms	618 ms	862 ms	1.2 s	1.6 s
Original (L2L)	91 ms	6.6 s	74 s	91 s	385 s	689 s
Blocks (L2L)	781 μs	2 ms	4 ms	8 ms	18 ms	57 ms
Convolution (L2L)	660 ms	2.0 s	3.4 s	5.0 s	5.5 s	5.8 s

Table 2 Execution times of a simulation of a single interaction between a source and a target of 20 years with hourly timesteps, using the original, the blocks, and the convolution methods, for several values of the tolerance ϵ and each of the considered cases: point to point, line to point, and line to line. In the convolution rows, the FFT algorithm has been used, and the tolerances refer to the adaptive quadrature tolerance used to compute the step responses.

Simulation method	$\epsilon = 10^{-2}$	$\epsilon = 10^{-4}$	$\epsilon = 10^{-6}$	$\epsilon = 10^{-8}$	$\epsilon = 10^{-10}$	$\epsilon = 10^{-12}$
Original (P2P)	5 ms	12 ms	24 ms	24 ms	36 ms	41 ms
Blocks (P2P)	18 ms	27 ms	32 ms	40 ms	49 ms	64 ms
Convolution (P2P)	4 ms	4 ms	4 ms	4 ms	4 ms	4 ms
Original (L2P)	6 ms	13 ms	24 ms	24 ms	36 ms	41 ms
Blocks (L2P)	18 ms	30 ms	47 ms	52 ms	77 ms	116 ms
Convolution (L2P)	4 ms	4 ms	4 ms	4 ms	4 ms	4 ms
Original (L2L)	6 ms	12 ms	24 ms	24 ms	36 ms	41 ms
Blocks (L2L)	22 ms	31 ms	57 ms	91 ms	121 ms	197 ms
Convolution (L2L)	4 ms	4 ms	4 ms	4 ms	4 ms	4 ms

Load aggregation and non-history acceleration methods comparison

This section intends to present similarities and differences, strengthts and weakenesses of the acceleration strategies investigated.

Both methods can be used to speed-up simulations and achieve suitable performance for borehole simulations. The load aggregation is an approachable method easy to understand, while the non-history acceleration method requires objectively more work to be grasped since the functions and variables arising from its formulation are not directly relatable to the physics of the problem. The authors believe that the work done in this project helped improving understanding and contribute in reducing this gap in accessibility between these two methods.

The two methods have a very different formulation, but when it comes to the actual computation at runtime they have a very similar structure. At each time steps in both cases there are two operations that needs to be done: the update of a vector containg a compressed version of the load, and the evaluation of the temperature of interest which is done in both cases via dot product of two vectors. For the load aggregation, the dot product is relative to the superposition of the effects and in the non-history method the dot product is relative to the evaluation of the non-history integral by numerical integration. As a result, the runtime of the two methods depends approximately on the size of the vectors used to perform the dot product for the two cases. In both cases, the size of the vectors utilized affects the error in the estimation but no direct comparison has been made yet to evaluate this aspect.

A difference between the two methods, is that the load compression strategy utilized in the non-history method does not affect the precision. Hence, with the non-history method (especially in the blocks variant) it is possible to achieve very high precisions (e.g. 10^{-10}) while keeping the size of the problem reasonable. Achieving high precisions might seems a little overkill for borehole applications. However, it is important to remember that the precisions discussed here are relative to single line-to-point or line-to-line temperature responses. In the context of borehole field simulation, where the thermal problem is coupled and the heat load distribution among the boreholes is affected by the local temperatures in the field, there can be certainly a compound effect that contributes in increasing the overall simulation error. It is therefore of interest having access to methods that can better control and estimate this error and that could be used as a reference to fine tune the precision on single source-to-target response that allows to achieve the desired precision in the overall borehole field simulation.

Simulation of complex borehole networks operation

The accelerations methods for time superposition have been integrated in the research softwares pygfunction, geothermsim, and BoreholeNetworksSimulator.jl. These advances in acceleration were crucial to achieve the simulation performance necessary to make complex long-term short time step simulations feasible.

In this section we present an example to showcase the functionalities that could be implemented in BoreholeNetworksSimulator.jl.

The model allows to perform hourly simulations of arbitrary borehole field configurations and piping network topology and perform long-term simulations with hourly steps. The software allows to reconfigure the network connections at any time during the simulation making it very flexible and suitable to explore operational strategies of large borehole fields.

The borehole network is connected to other systems through a manifold piping. The boundary condition at the manifold can be either given flow rate and overall power exchanged or given flow rate and inlet secondary fluid temperature. The latter condition is of particular interest because it allows to explore how much heat is possible to extract or inject from the system when supplying a given temperature and flow which is the realistic condition in most practical cases.

To showcase the potential of the model, let's consider the borehole configuration in Figure 14. The borehole field comprises 48 boreholes and the manifold supplies 8 loops with 6 boreholes in series. The borehole length is 75 meters, the ground conductivity is 3 W/(m K), the ground thermal diffusivity is 4.8 * 10^{-7}, and the flow rate supplied is 0.5 kg/s.

The borehole field is operated 6 months with injection temperatures of 90 °C and 6 months with extraction temperature of 55°C. Injection is performed by circulating from the inner boreholes to the outer boreholes, while extraction is obtained by reversing the flow direction and circulating from the outer borehole towards the inner boreholes. Finally, the system is operated intermittently and works only 12 hours per day.

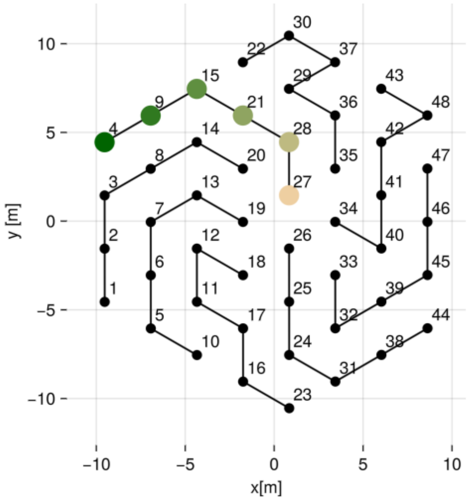


Figure 14 Exanple of borehole network configuration. Boreholes linked by segments are connected in series. All series loops are connected in parallel. The borehole on the loop 4-9-15-21-28-27 are color coded. The same colors are used in the following plots to display power and temperatures relative to these boreholes.

The results of a 10-years hourly simulation are displayed in the following figures. The plots display fluid inlet temperature, borehole temperature and power exchanged for a borehole loop comprising borehole 4,9, 15, 21,28 and 27. The plots shows how the charging and discharging strategy promotes stratification in the borehole field, e.g. the temperature is higher in the inner boreholes and lower on the periphery. Stratification in storage is important to limit heat losses and preserve higher temperature heat (higher quality energy) in the center of the of the storage.

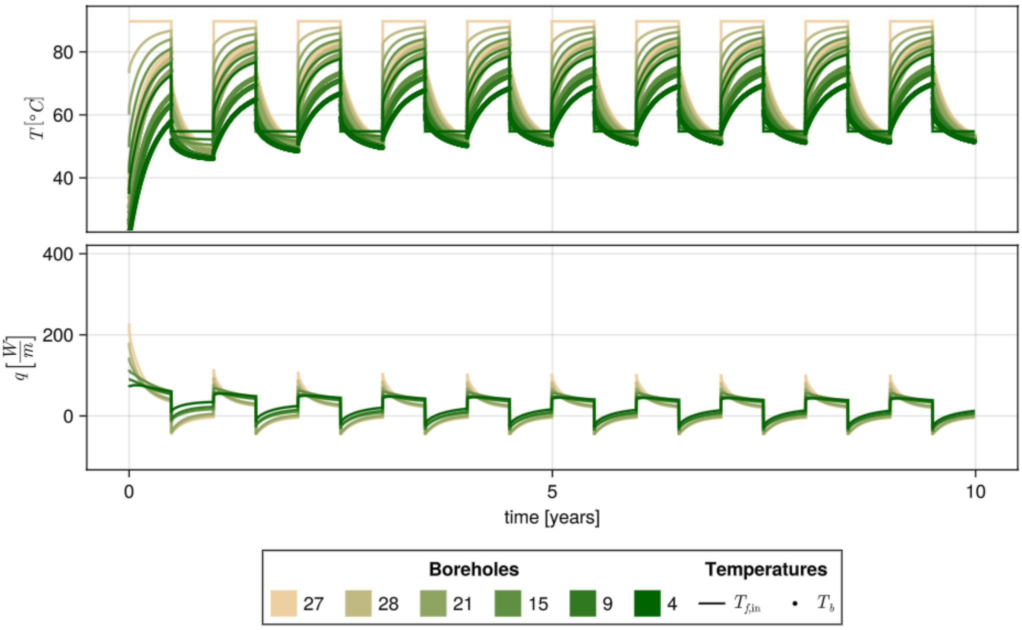


Figure 15 Mean daily power and temperature exchanged on the test borehole loop during the whole simulation period. The values are averaged for visualization purpose.

Figure 16 shows the results during a week in the middle of the injection season at the 10th year of operation. It is possible to observe that injection heat rates are higher at the periphery than in the center, due to increase in temperature in the center and the reduced potential for heat exchange in this area.

Figure 17 shows the results during a week at the beginning of the extraction season during the 10th year of operation. The potential for extraction on the periphery is limited, and the internal boreholes extract significantly higher heat rates compared to the external boreholes.

Figure 18 shows the results during a week at the end of the extraction season during the 10th year of operation. For borehole 4 the temperature dropped below the injection temperature of 55°C and the circulation results in injection into this specific borehole. In general, the temperature has dropped significantly due to both extraction and heat diffusion in the rock, and the potential for extraction at 55°C is highly reduced compared to the beginning of the season.

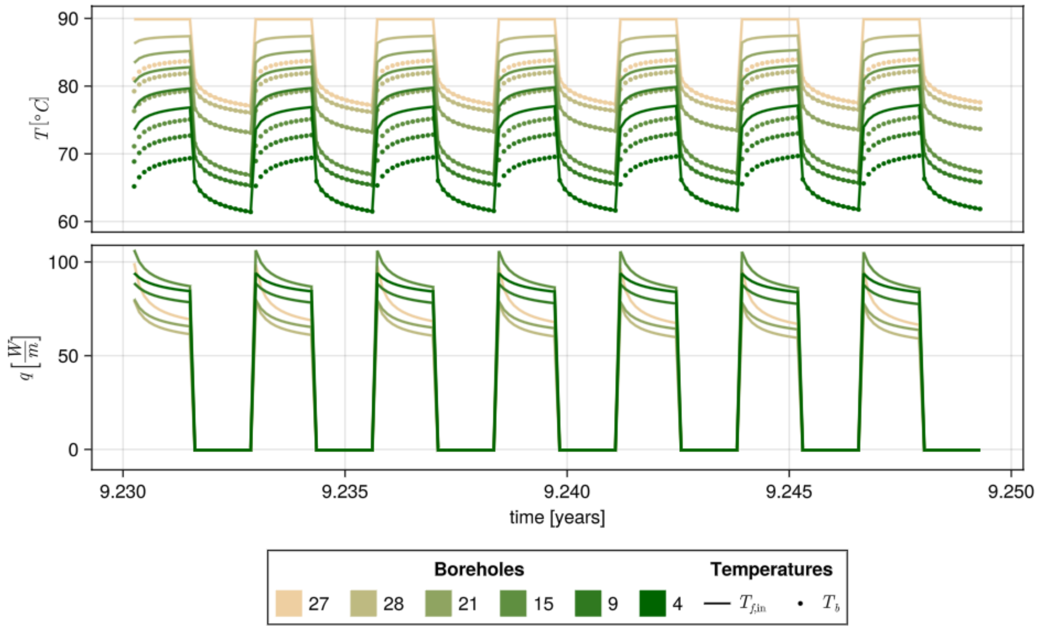


Figure 16 Hourly power and temperature exchanged on the test borehole loop during a week in the middle of the extraction season on the 10th year of simulation.

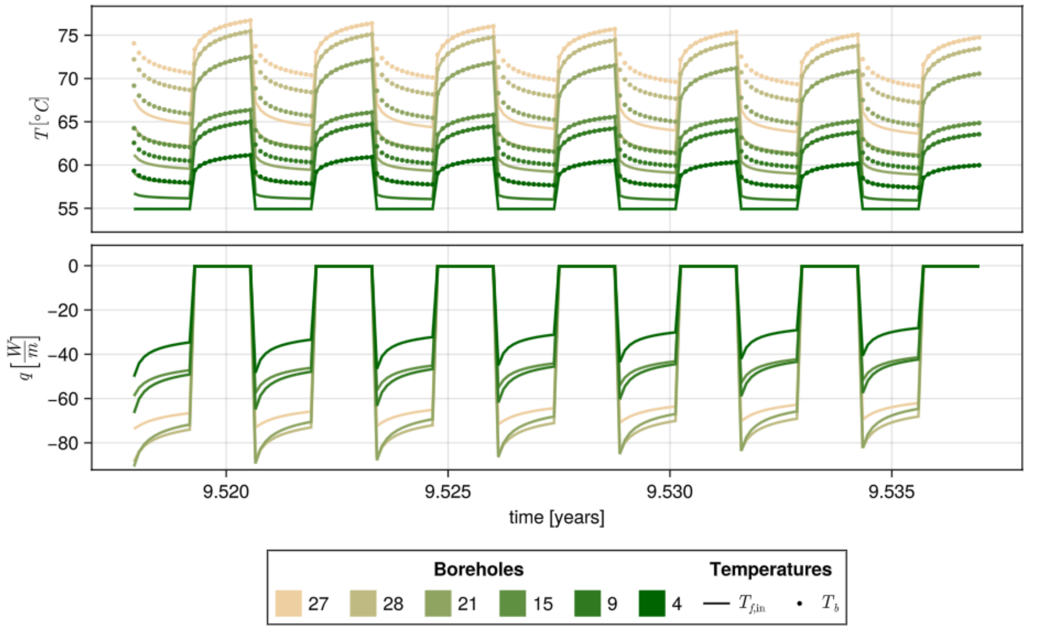


Figure 17 Hourly power and temperature exchanged on the test borehole loop during a week at the beginning of the extraction season on the 10th year of simulation.

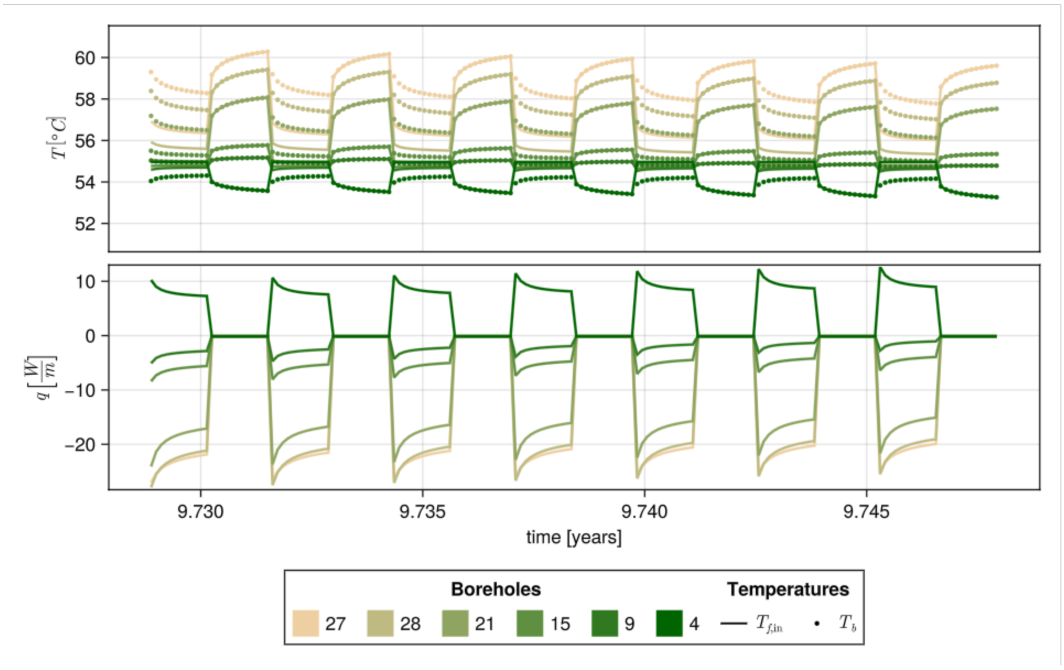


Figure 18 Hourly power and temperature exchanged on the test borehole loop during a week at mid extraction season on the 10th year of simulation.

The example shows the flexibility of the tool and its ability to showcase how the network operation affects heat distribution and temperature in the borehole field.

Thanks to the acceleration algorithms implemented, the 10 years hourly simulation is performed by the simulator in around 40 seconds which is a satisfying result in terms of performance. This efficiency makes the model suitable to test several operational strategies and optimize borehole heat storage control.

Discussion

This work allowed to achieve very significant fundamental results in terms of improvement of acceleration algorithms. This development was critical to expand the capabilities of borehole model based on semi-analytical methodologies and make them viable tools for long term prediction of complex operations of borehole fields while simulating the system with at fine time resolution.

The algorithmic improvements achieved allowed to reduce the computational time so that complex borehole network simulations can now be perfomed on laptops in minutes. These improvements were implemented in open source borehole simulations making the developments available to end users in the field of geothermal systems.

We believe that it is important to continue developing tools that can help improve planning and integration of geothermal system in large thermal networks. Integration is a challenging engineering task and specialized tools to aid decision making are needed. The efficiency and accuracy of the semi-analytical methods investigated in this work presents a lot of advantages against other modeling approaches. This fact makes them very usefull for engineering applications and for their integration with modeling tools for building simulations or energy systems simulations. In particular, the fact that it is possible to run hourly simulation for long term makes this modeling strategy a good candidate for integration since hourly simulations are standard in buildings and energy system simulations.

As showed in the example, the models allow to simulate fairly complex problems in short time. However, there are some limitations in the set of problems that can be currently solved at the moment, and furhter development is needed to address them. One relevant limitation is that it is currently not possible to accurately represent a finite insulation layer boundary condition on the top surface of a storage. This condition is relevant for high temperature storage systems where it is normally desirable to install insulation in practice to avoid excessive thermal losses. The boundary condition that can be currently simulated are either a constant uniform temperature boundary condition or an adiabatic boundary condtion (perfect insulation). Future work is needed to address this limitation and to make the results obtained with these models even more reliable and realistic also for high temperature applications.

In conclusion, the geothermal modeling development carried out in this project is a stepping stone to provide accurate and fast long-term hourly simulation of borehole networks operation which will serve scientists and engineers to optimize borehole fields operation and integration within the energy system.

Publication list

M. Cimmino, A. Lazzarotto. Load aggregation for thermal interactions between boreholes. *IGSHPA Research Conference Montréal*, May 28-30, 2024. https://doi.org/10.22488/okstate.24.000010

A. Lazzarotto, M. Basquens, M. Cimmino. A non-history dependent temporal superposition algorithm for the point source solution, *IGSHPA Research Conference Montréal*, May 28-30,2024https://doi.org/10.22488/okstate.24.000021

A. Lazzarotto, M. Basquens, M. Cimmino. A non-history dependent temporal superposition algorithm for the finite line source solution. *Submitted to Journal*. Preprint available https://doi.org/10.48550/arXiv.2507.18200

M. Basquens, A. Lazzarotto. A causality inspired acceleration method for the fast temporal superposition of the finite line source solutions. *Submitted to Journal*. https://doi.org/10.48550/arXiv.2507.18200

M. Cimmino, M. Basquens, A. Lazzarotto. Higher-order semi-analytical model for the simulation of geothermal boreholes. I*nternational Journal of Thermal Sciences*. Volume 219, January 2026, 110184.

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Attachments

- *Administrativ bilaga*